The party decides the rules: How different vote counting rules change the outcome of the 2016 Republican Primary *

Adam M. Dynes†
Brigham Young University

Thomas J. Wood‡
The Ohio State University

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Abstract

Social choice theory suggests that decision rules used to aggregate voters' preferences (e.g., plurality, instant-runoff, Borda count, or approval voting) form the basis of electoral choice in elections and can affect electoral outcomes. We use individual-level data from Republican voters, including several key primary states in the 2016 Republican Presidential primary, to show how different vote counting rules dramatically change the outcomes in these races. For example, even though Donald Trump won a plurality in New Hampshire, we find that he would have placed 4th, on average, under a wide range of alternative vote counting rules. Our findings have important implications for how we interpret the meaning of Donald Trump's 2016 nomination. In addition, political scientists who have focused on the important role that party elites play in coalescing around a favored nominee in terms of endorsements, financing, and other traditional campaign tools, have not focused much on how those same party elites could reform their electoral systems to help rank-and-file party members unify behind a consensus candidate.

*THANKS TO COME.

†Assistant Professor, Department of Political Science; adamdynes@byu.edu

‡Assistant Professor, Department of Political Science; wood.1080@osu.edu

Donald Trump was a uniquely un-endorsed candidate in contemporary presidential primaries. And he didn't just lack endorsements from other Republican officials; he also lacked broad support among Republican voters. In a survey taken just before the 2016 Republican convention, 52% of Republican respondents indicated that they "wish[ed] someone else were the nominee."[1] Though Trump's unlikely nomination may seem to be an aberration (at least in presidential primaries), the nominating systems employed by the two major political parties at nearly every office level are actually quite susceptible to the problems his nomination illustrates (e.g., on Congressional primaries, see Bawn et al. 2014). Most political observers realize that increased democratization of parties' nominating systems diminishes the ability of party elites to influence who wins. In a somewhat ironic twist, this increased democratization as currently practiced may also diminish the ability of voters to coalesce around consensus candidates, resulting in the nomination of candidates who are unsatisfactory to both party elites and the masses.

This problem is well-known among scholars of social choice theory, and yet its implications for political parties' nominating systems is often overlooked by both political practitioners and observers. In the wake of the 2016 Presidential primaries, elites in both parties have proposed reforms[2] to avoid future primaries in which their preferred candidates are threatened by outsiders, like Donald Trump and Bernie Sanders, who disrupt the party's coalition and lack broad support among party voters. However, most reforms discussed by party elites to achieve these ends, such as restricting who can participate in primaries or giving elected officials greater influence as "super delegates," are anathema to party regulars. At the same time, political scientists who study the role of party elites in primary elections focus on how elites use endorsements, donations, and other traditional campaign tools to influence outcomes (Cohen et al. 2009; Hassell 2016, 2017), ignoring how those same party elites could reform their electoral systems to allow rank-and-file party members to identify consensus candidates.

What many political observers overlook is a more feasible and less controversial solution to this problem—change the voting rules used to aggregate voters' preferences in ways that favor candidates who have broader support among voters. Most political parties use a plurality voting rule, giving voters one vote to cast. In situations where there are multiple candidates who are generally supported by a majority of voters, these

[1] Gallup, "Less Than Half of Republicans Pleased With Trump as Nominee", August 19, 2016, accessed March 15, 2017, http://news.gallup.com/poll/194738/less-half-republicans-pleased-trump-nominee.aspx

[2] https://newrepublic.com/article/130396/broken-presidential-nominating-system

candidates may split the votes of the majority, allowing a candidate with less support to win the nomination.[3] To highlight this possibility, in this paper we demonstrate how different voting rules (e.g., plurality, instant-runoff, Borda count, approval voting, etc.) could have potentially changed the outcome in early contests in the 2016 Republican presidential primaries.

To do so, we use individual-level data from Republican voters in Iowa, New Hampshire, South Carolina, and Nevada—the first four states that held primaries or caucuses in 2016. Our results suggest that the Condorcet winner (i.e., the candidate who could beat all other candidates in a pairwise comparison) did not win in two of the four states—New Hampshire and South Carolina. In addition, we find that different candidates would have won in those two states under several different voting rules—top-two runoff, Borda, and Black. If these other rules were implemented instead of the current practice of determining "winners" by plurality rule, we predict that Trump may have only won in one state (Nevada) instead of three (Nevada, New Hampshire, and South Caroline). In addition, his performance in Iowa would have also been diminished under pairwise comparisons, where he finished fourth instead of second. Given the role of momentum in presidential primaries (Bartels 1988), these different electoral outcomes in early primary states could have changed the course of the remaining primaries, leading to the nomination of a candidate with broader support among both party elites and voters.

Though our analysis focuses on the 2016 Republican presidential primary, the findings and their implications are broadly applicable to primaries across the US. In fact, situations like the 2016 Republican presidential primary, in which a less broadly supported candidate wins the nomination, may occur much more regularly in primaries for lower level offices than in those for the presidency. For example, Bawn et al. (2014) find that most open-seat primaries for winnable Congressional seats feature more than two viable candidates. In 63% of these primaries in 2013 and 2014, the second place finisher could have won if they had received the votes that went to the other trailing candidates. In other words, the possibility for spoilers is quite large. Primaries in which non-consensus candidates win may also be less common in presidential elections because of the attention given to these elections by party elites. With the high stakes involved

[3]Though many state parties award presidential convention delegates proportionally, they still limit voters to a single vote. This problem also applies to state parties that use run-off elections, which is the case for most Southern states in Congressional primaries. Though such systems can increase the probability that consensus candidates win, they are still susceptible to the possibility that these candidates do not proceed past the first round of votes.

in controlling the White House, party elites have strong incentives to develop tools to influence the outcome of their respective primaries (Cohen et al. 2009). Though there is evidence that party elites use similar tactics in Senatorial primaries (Hassell 2016, 2017), the research on primaries for lower level offices is generally lacking. In addition, we lack detailed public opinion data in many of these races to evaluate 1) the extent to which there is broad support for primary winners among primary voters and 2) how primary outcomes would change under different vote aggregation rules. The available evidence, however, suggests that our findings apply to many primaries in the US.

These findings have several important implications for our understanding of political parties and presidential primaries. First, as mentioned already, they highlight a class of reforms that could be implemented in nominating systems to increase the probability of nominating consensus-building candidates without curbing the high levels of democracy currently found in the parties' nominating systems. Such reforms may be more acceptable to the parties' rank-and-file who are critical of the other reforms suggested by party elites to stop candidates who lack broad support among voters and elites.

Second, these findings illustrate the applicability of social choice theory to a major election, even one in which elites have developed institutions to avoid some of the problems identified by social choice theory in elections with multiple candidates. Third, the findings also change how we should interpret the meaning of Trump's nomination. Many political observers and officials, including Donald Trump, claim that his nomination means that Republican voters support him or his agenda. Besides ignoring the fact that voting choices are often multi-dimensional, these claims of electoral "mandates" ignore how voting decision rules used in presidential primaries affect who wins. Though this important finding from social choice theory is well known among social scientists, its implications for interpreting public opinion as expressed through elections is often ignored, especially among non-scholars.

Finally, our findings also have a simple implication for the news media and polling firms in races that feature three or more viable candidates. Rather than just have voters indicate their top choice, surveys should also ask respondents to (at least) rank order a few of the candidates so that we can have a better information about the voters' collective preferences.

1 Different Types of Voting Rules

The argument and finding that vote aggregation rules affect electoral outcomes is well known in the social sciences (Rae 1967; Brams and Fishburn 1978, 2007; Riker 1982). Much of the work on in this literature focuses on the welfare implications of these different rules. By "welfare implications," we refer to whether certain rules are more or less likely to meet specific criteria that are deemed desirable (Condorcet 1785; Arrow 1951; Harsanyi 1986) or result in voting paradoxes (Plassmann and Tideman 2014; Gehrlein, Lepelley et al. 2017) or strategic voting[4] (Merrill 1984; Green-Armytage, Tideman and Cosman 2016). A common finding in these analyses is that the voting rule used in nearly all US elections—i.e., plurality or "first-past-the-post"—performs quite poorly relative to other voting rules in situations with more than two candidates, which is why social choice theorists often favor other types of voting rules in electoral settings, like primaries, that often feature multiple candidates (Black et al. 1958; Brams and Fishburn 1978, 2007; Felsenthal 2012).

Before reviewing the performance of different voting rules across a variety of metrics, we summarize how winners are chosen by some commonly studied rules that will also be examined here in this analysis. The descriptions for each rule, except approval voting and top-two runoff, come directly from Plassmann and Tideman (2014) and assume that voters rank order candidates from their most preferred to least preferred on their ballot. Such rank ordering is not necessary with the plurality rule, approval voting, or top-two runoff if the latter consists of two rounds of voting.

We begin with rules that do not require rank ordering in the ballot:

- **Approval Voting**. Voters select as many or as few candidates on the ballot as they desire. Count how many voters selected each candidate. The candidate selected the most wins.

- **Plurality** (also known as **First Past the Post)**. Count the number of times each candidate is ranked first. The candidate with the largest number of first ranks wins. In practice, this voting rule does not require voters to rank order candidates

[4]Strategic voting is also known as insincere voting, tactical voting, or sophisticated voting. It occurs when a voter does not reveal their true rank ordering of candidates on their ballot in order to increase the likelihood of an election outcome that they prefer over what would occur if they cast a sincere vote. In plurality voting, this would consist of voting for someone other than one's most preferred candidate in order to prevent the election of some other candidate that the voter prefers even less.

on their ballot. Rather, voters have one vote and just select one candidate among all available candidates.

- **Top-Two Runoff** (also known as **Plurality with Runoff)**. Count the number of times each candidate is ranked first. Identify the two candidates with the largest and second largest number of first ranks wins. Eliminate all other candidates. Count the number of times that each of the remaining candidates is ranked higher than the other. The candidate ranked higher the largest number of times wins. In practice, this rule is implemented in two rounds of voting. In the first round, the first and second place finishers under the plurality rule move on to the second round of voting. In the second round, the winner is chosen based on the plurality rule.

We now list rules that require rank-ordering but are not Condorcet-consistent:[5]

- **Borda**. For each ballot, assign $m - 1$ points to the candidate ranked highest, $m - 2$ points to the candidate ranked second highest, and so on (where m is the number of candidates). A candidate's Borda score is the sum of the points calculated for the candidate over all ballots. The candidate with the highest Borda score wins.

- **Coombs** (also known as **Negative Plurality Elimination**). If there is a candidate who is ranked first by a majority of the voters, that candidate wins. If not, then eliminate the candidate ranked last on the most ballots and rewrite the ballots without this candidate. Continue this elimination process until one candidate has a majority of the first-place votes. That candidate wins. [PLEASE NOTE: In this current version of the paper, we do not analyze who would win in the 2016 Republian primaries under the Coombs rule. However, we plan to include this voting rule in the analysis in subsequent versions of this paper.]

- **Hare** (also known as the **Alternative Vote**, **Instant-Runoff Voting**, **Plurality Elimination**, or the **Single Transferable Vote**[6]). If there is a candidate who is ranked first by a majority of the voters, that candidate wins. If not, then eliminate the candidate ranked first by the fewest voters and rewrite the ballots without this

[5]Condorcet-consistent rules select the Condorcet winner as the candidate if one exists. A Condorcet winner is a candidate who would win a plurality vote against each of the other candidates in a two-candidate election.

[6]This is only the case in elections where a single candidate is elected as opposed to multiple.

candidate. Continue this elimination process until one candidate has a majority of the first-place votes. That candidate wins.

Finally, here is a well-known rule that requires rank-ordering and is Condorcet-consistent:

- **Black**. If there is a candidate who is a Condorcet winner[7], then that candidate wins. If not, then the candidate with the highest Borda score wins.

2 Which Voting Rule is "Best"?

As mentioned, a common goal of research on these and other different voting rules is to examine their optimality based on a variety of criteria. Unfortunately, there is no consensus on which voting rule is the best one as all have different strengths and weaknesses. All voting rules violate at least one of the minimal criteria laid out by Arrow's Impossibility Theorem (Arrow 1951). They are also all susceptible to at least one or more voting paradoxes (Felsenthal 2012; Plassmann and Tideman 2014; Gehrlein, Lepelley et al. 2017) and usually susceptible to strategic voting (Green-Armytage, Tideman and Cosman 2016). They also place different burdens on voters and election administrators in terms of political knowledge and their effects on the probability of voters' spoiling their ballots (Felsenthal 2012, 34). Though it is beyond the scope of this paper to establish which voting rule(s) would be ideal for US presidential primaries, readers should be somewhat aware of these issues as we analyze how the different Republican candidates in the 2016 primary would fare under different voting rules. If different voting rules result in different winners, the desirability of implementing that rule instead of another would depend on a variety of factors and normative goals. Nevertheless, the literature consistently finds that plurality voting, the most common voting method in US elections, rates poorly across a wide range of criteria, which we review briefly below.

In terms of election paradoxes, there are two that are well known and considered important to avoid. The first is the Condorcet winner paradox. As Plassmann and Tideman (2014) explain: "A (strong) Condorcet winner is a candidate who beats all other candidates in pairwise comparison, using majority rule. The Condorcet winner paradox arises when there is a Condorcet winner but a different candidate is declared the winner" (36). Under the normative assumption that democratic processes should result in

[7] A Condorcet winner is a candidate who would win a plurality vote against each of the other candidates in a two-candidate election.

outcomes favored by the majority, nominating a Condorcet winner when one exists is considered "the weakest extension of the majority rule principle" (Felsenthal 2012, 35; see also Black et al. Black et al. and Green-Armytage 2004), which is why rules are often evaluated in terms of their susceptibility to the Condorcet winner paradox (Merrill 1984; Felsenthal 2012; Plassmann and Tideman 2014; Gehrlein, Lepelley et al. 2017).

A somewhat related paradox that is considered even more problematic (Felsenthal 2012) is the Condorcet loser paradox (also known as the Borda paradox). Again from Plassmann and Tideman (2014): "A Condorcet loser is a candidate who loses against all other candidates in pairwise comparison, using majority rule. The Condorcet loser paradox arises when there is a Condorcet loser, and the Condorcet loser is declared the winner" (36). Approval voting, plurality, and top-two runoff are susceptible to both of these paradoxes while the alternative vote, Borda, and Coombs are only susceptible to the Condorcet winner paradox (Felsenthal 2012). By design, the Black rule is not susceptible to either (Felsenthal 2012), which is why Felsenthal (2012) favors it over the others.

The degree to which a voting rule is hampered by voting paradoxes and manipulation depends on a variety of factors. For instance, as the number of candidates in an election increases (Merrill 1984) or the number of voters decreases (e.g., below 100 such as in legislative bodies) (Plassmann and Tideman 2014; Felsenthal, Maoz and Rapoport 1993), the probability that a paradox will occur increases under a voting rule that is susceptible to such a paradox. Using simulated data of voters and candidates arrayed in a two-dimensional policy space, Merrill (1984) finds that the best to worst rule in terms of electing the Condorcet winner when one exists are Black, Coombs, Borda, Hare, top-two runoff, approval voting, and plurality. Plassmann and Tideman (2014), who use surveys of voters' preferences in actual elections to simulate three-candidate races, come to similar conclusions as Merrill (1984) in terms of these rules' susceptibility to the Condorcet winner paradox as well as a variety of other paradoxes. In terms of maximizing social utility under the assumption that voters receive more utility when a candidate closer to their ideal point is elected, Merrill (1984) finds that Borda, Black, and approval voting perform best while plurality performs the worst. Several other scholars come to similar conclusions (Weber 1977; Bordley 1983; Green-Armytage, Tideman and Cosman 2016) though Bordley (1983) would rate approval voting higher than Borda in terms of overall desirability. Finally, in terms of susceptibility to manipulation from strategic voting, Borda, Coombs, and plurality perform quite poorly overall while Hare performs

quite well (Merrill 1984; Chamberlin 1985; Lepelley and Valognes 2003; Green-Armytage, Tideman and Cosman 2016).

In sum, different voting rules outperform each other across these different metrics. However, plurality voting consistently performs at the bottom, being the most susceptible to voting paradoxes and strategic voting while being the least likely to maximize social utility. Moreover, its performance on these metrics declines as the number of candidates increases. These findings hold in studies that examine actual election data to identify the degree to which these different voting rules are prone to undesirable outcomes. A major shortcoming of the literature on voting rules is the lack of systematic empirical data from actual elections that can illustrate how often these undesirable outcomes occur (Felsenthal, Maoz and Rapoport 1993; Gehrlein 2006). Nearly all of the findings cited above are based on theoretical proofs or simulated data. If these paradoxes are just possible but rare, then concerns about the type of voting rule used may be overstated. Some of the best empirical data on this topic comes from a database of actual ballots from 92 elections held by unions and professional societies in the UK. Voters rank-ordered candidates in these elections, which also varied significantly in terms of the number of candidates (from 3 to over 20) and voters (from 32 to over 3,000). Consistent with the theory and data simulations, analyses of these rank-ordered ballots find that plurality performs worst than other voting rules (Felsenthal, Maoz and Rapoport 1993; Felsenthal and Machover 1995) though differences in the performance of rank-ordering voting rules are generally small.

3 Empirical Studies of Voting Rules

Systematic empirical data from actual elections is lacking mostly due to the fact that the vast majority of elections do not use rank-ordering voting rules. This reduces researchers' access to actual ballots with voters' preference ordering. It also means that most surveys of voters' electoral preferences only ask respondents to indicate the candidate that is their first and perhaps second choice in the election.[8] Gehrlein (2006, see sections 2.2 and 2.3) surveys many of the empirical studies on this topic, with a partic-

[8]One exception is the American National Election Studies (ANES), which frequently has respondents rate candidates for US presidential election on a 101-point feeling thermometer. We discuss these in more detail below.

ular focus on studies that measure the frequency that the Condorcet paradox[9] occurs. Many of these examine legislative votes as opposed to elections (e.g., Riker 1982; Jenkins and Munger 2003). So far, we have identified about a dozen studies that use data from voters in actual elections for public office. The contexts include elections for the US president (Riker 1982; Abramson et al. 1992; Brams and Merrill 1994; Radcliff 1994; Abramson et al. 1995; Regenwetter, Adams and Grofman 2002; Regenwetter, Grofman and Marley 2002; Tideman and Plassmann 2012), Danish Prime Minister (Kurrild-Klitgaard 2001), national parliaments in several European countries (Van Deemen and Vergunst 1998; Regenwetter, Adams and Grofman 2002), US Senate (Taylor 1997), and mayors in two cities (Dietz and Goodman 1987; Hsieh, Emerson and Paolino 1997). Overall, these articles find that voting paradoxes occur in some but not all of these elections and that results could change slightly under different voting rules, though this is not the case in general elections for US presidents.

Previous studies on voting rules and US presidential elctions differ substantially from ours in several important ways. First, they primarily focus on the general election as opposed to the primary election. This includes Brams and Merrill (1994), who are motivated by a question similar to ours.[10] Two of these studies (Abramson et al. 1992; Radcliff 1994) also include voters' preferences over a few candidates in the primary, but the total number of candidates from the same party is still relatively low with three at most. As previous work has found (Merrill 1984), voting paradoxes are more likely to occur in elections where there are more candidates and where voters' preferences over those candidates are similar. These conditions are much less likely to be met in general elections, where voters have more clearly differentiated preferences over candidates. Our analysis, on the other hand, involves voters' preferences over 6 candidates all in the same party's primary, a scenario in which voting rules are much more like to affect the outcome.

Our paper also differs from past work, except for Brams and Merrill (1994), in terms of the primary question we address. For example, Abramson et al. (1992) and Abramson et al. (1995) focus on the extent to which strategic voting occurred in the general election while Radcliff (1994) as well as Abramson et al. (1995) focus on whether the Condorcet

[9]The Condorcet's Paradox (also known as cycling) "occurs when there is no candidate who beats all other candidates in pairwise comparisons using majority rule" (Plassmann and Tideman 2014, 39). In other words, these are elections in which collective preferences are nonstransitive.

[10]They examine whether the 1992 presidential election results would have changed under approval voting since there was a strong third party contender. Their answer is "no."

winner ultimately won. The other three studies (Regenwetter, Adams and Grofman 2002; Regenwetter, Grofman and Marley 2002; Tideman and Plassmann 2012) use data of voters' preferences in US presidential elections to examine modeling assumptions used in analyses of voting rules that rely on simulated data.

4 Survey Data of Republicans in Early Primary States

To examine how different voting rules would change the outcomes in the 2016 Republican primary, we analyze individual-level survey data of likely Republican voters in the first four states that had a primary or caucus in 2016—Iowa, New Hampshire, South Carolina, and Nevada. The key is to identify survey data where respondents rank order or rate the candidates in some manner that likely reflects their vote choice preference ordering. Unfortunately, nearly all surveys of primary voters only ask respondents to indicate the one candidate for whom they plan to vote, making it impossible to estimate their preferences over the candidates beyond their first choice.

To overcome this problem, we identified a series of surveys that asked respondents to evaluate multiple Republican candidates using a 101-point feeling thermometer where low scores are supposed to indicate unfavorable feelings toward a candidate and high scores indicate favorable ones. Previous work on voting rules and paradoxes in US presidential elections uses this same measure as an indication of vote choice (Abramson et al. 1992; Brams and Merrill 1994; Radcliff 1994; Abramson et al. 1995; Regenwetter, Adams and Grofman 2002; Regenwetter, Grofman and Marley 2002; Tideman and Plassmann 2012). The assumption is that voters' rank ordering of candidates based on their feeling thermometers scores reflect their rank ordering of the candidates in an actual vote. In other words, our analysis and previous ones assume that voters' first choice in the primary is the candidate whom they rate highest on a feeling thermometer, and their second choice is the candidate whom they rate second highest, and so forth. (We examine this assumption in the next section.)

The surveys were administered by a survey firm working for one of the Republican presidential candidates to assess public opinion among likely Republican voters in the first four states holding a primary or caucus. The surveys were conducted in live telephone interviews over landlines and cell phones with a sample of nearly 1,600 respondents from each state. The surveys in Iowa and New Hampshire were fielded from November 30 through December 15, 2015 while the surveys in South Carolina and

Nevada were fielded from January 4 through 13, 2016. All of the surveys were conducted about a month or two before the primaries or caucuses took place in these states (Iowa - Feb. 1; New Hampshire - Feb. 9; South Carolina - Feb. 19; Nevada - Feb. 22).[11] In earlier surveys of likely Republican primary voters, the survey firm had realized that a high percentage of respondents were still undecided in their vote choice and therefore chose to exclusively use feeling thermometers in these surveys to gauge respondents' preferences over candidates.

Given time and budget constraints, the survey firm asked respondents to rate only six Republican candidates (as opposed to the full field of 17) using a 101-point feeling thermometer where "0 means that you feel very unfavorable and 100 means you feel very favorable toward that person." In figure 1, we display violin plots of respondents' ratings of the candidates they were asked to rate in each survey in each state. Thick parts of the distribution indicate areas of the thermometer where there are more respondents. The labelled points indicate the mean feeling thermometer ratings for each candidate.

To further help readers assess the distribution of the feeling thermometer ratings, we present some basic descriptive statistics of the ratings in table 1. The statistics include each candidate's mean rating on the thermometer rating as well as the percent of respondents who gave the candidate either highly favorable ratings (at or above 80) or highly unfavorable ones (at or below 20). These numbers also help us evaluate the extent to which preferences over candidates were bimodal, which has implications for candidates' performance in a voting system where voters can only cast one vote. The difference in table 1 is the percent of respondents who gave the candidate a score at or above 80 minus the percent who gave him a score at or below 20. Finally we also list the Bimodal Coefficient for each candidate which measures the extent to which a distribution is bimodal (Pfister et al. 2013). Distributions with a coefficient above 0.555 are considered to be bimodal.

Several interesting patterns emerge from these feeling thermometer ratings. First, it is clear that Kasich, Bush, and Christie were the least popular of the candidates across the states except for in New Hampshire, where Bush was still the least favored candidate but the distributions between candidates are quite similar. A second interesting pattern is that based on mean ratings, Trump runs in the middle of the pack behind Cruz, Rubio, and Carson in all of the states but Nevada. However, he had the highest vote share in

[11] If readers are aware of surveys conducted closer to these primaries that also have some sort of rank ordering data, please let us know so that we can include it in our analysis.

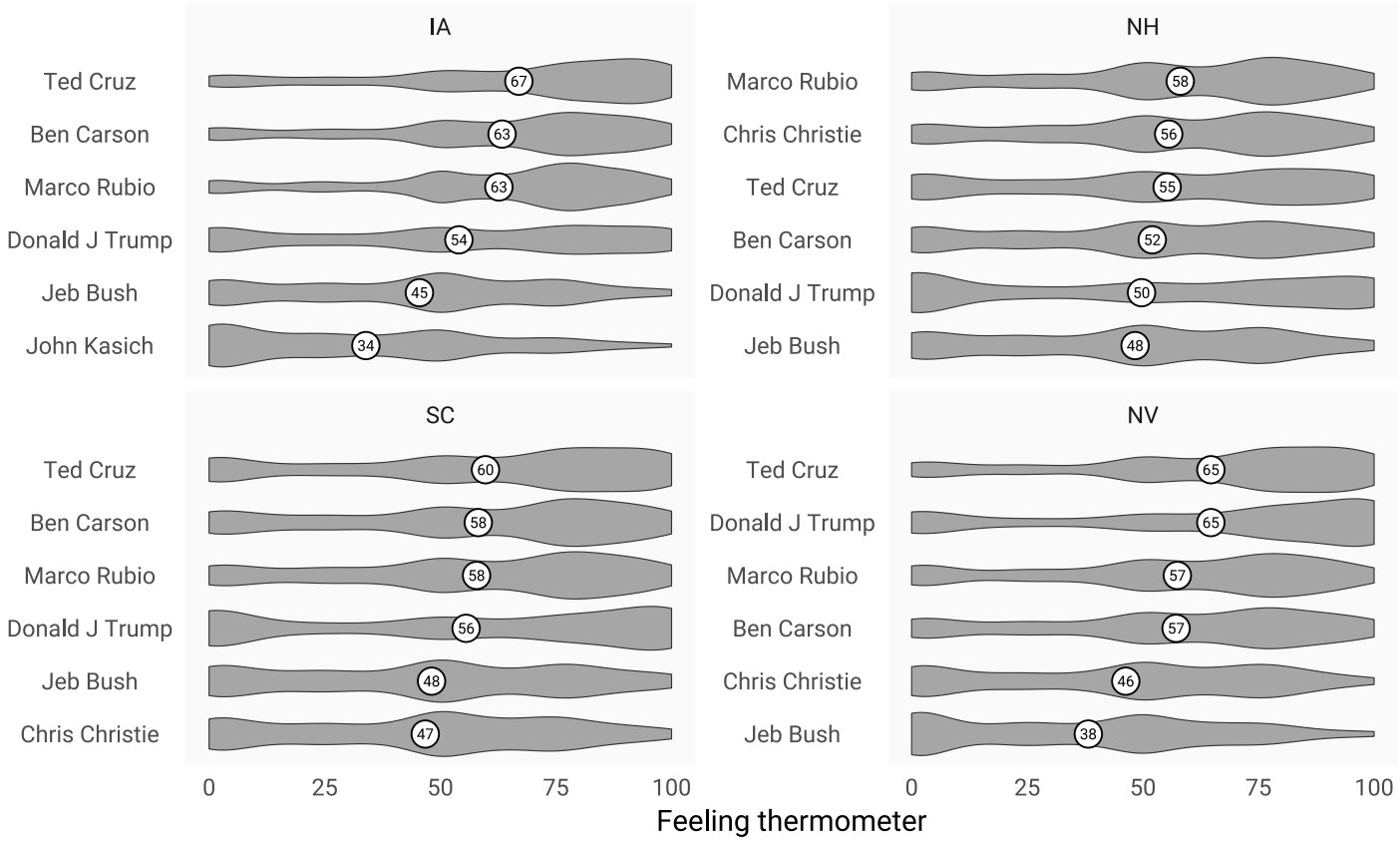

Figure 1: **Feeling Thermometer Ratings of Republican Candidates by State**. Each panel shows violin plots of likely Republican primary voters' ratings of Republican candidates in the 2016 Republican Primaries using a 101-point feeling thermometer, where 0 indicates very unfavorable feelings, and 100 indicates very favorable feelings. Thick parts of the distribution indicate areas of the thermometer where there are more respondents. The labelled points indicate the mean feeling thermometer ratings for each candidate.

	Candidates						
State	Trump	Cruz	Rubio	Carson	Christie	Bush	Kasich
Iowa							
Mean Rating	54	67	63	63	–	45	34
% at or above 80	33%	48%	36%	39%	–	13%	8%
% at or below 20	24%	12%	11%	12%	–	26%	42%
Difference	10	36	25	27	–	-13	-34
Bimodal Coeff.	.62	.32	.29	.29	–	.50	.23
New Hampshire							
Mean Rating	50	55	58	52	56	48	–
% at or above 80	33%	32%	31%	24%	27%	18%	–
% at or below 20	33%	22%	15%	22%	17%	24%	–
Difference	0	10	16	2	10	-6	–
Bimodal Coeff.	.69	.23	.25	.22	.24	.53	–
South Carolina							
Mean Rating	56	60	58	58	47	48	–
% at or above 80	39%	39%	33%	34%	16%	21%	–
% at or below 20	28%	19%	17%	19%	25%	25%	–
Difference	11	20	16	15	-9	-4	–
Bimodal Coeff.	.70	.27	.25	.25	.21	.21	–
Nevada							
Mean Rating	65	65	58	57	47	38	–
% at or above 80	50%	44%	32%	31%	17%	10%	–
% at or below 20	19%	14%	17%	18%	27%	35%	–
Difference	31	30	15	13	-10	-25	–
Bimodal Coeff.	.72	.30	.26	.25	.21	.21	–

Table 1: **Descriptive Statistics of Feeling Thermometer Ratings for Each Candidate by State**. This table displays some basic descriptive statistics of likely Republican primary voters' ratings of each candidate on a 101-point feeling thermometer where 0 indicates a very unfavorable feeling toward a candidate and 100 indicates a very favorable feeling. The first statistic is each candidate's meaning rating on this measure followed by the percent of respondents who either gave a candidate very high scores (at or above 80) or very low scores (at or below 20). We then show the difference between these numbers. The final statistic, the Bimodal Coefficient, indicates the extent to which the feeling thermometer ratings exhibit a bimodal distribution. Values above 0.555 are considered to indicate a bimodal distribution.

three of the states and the second highest in one (Iowa). This suggests that the distribution of preferences over Trump was more bimodal than it was for other candidates, a pattern which is confirmed both visually in figure 1 and according to the statistics in table 1. In the first three states, where Trump's mean favorability rating is below the median, the percent of voters who give him a high favorability score (at or above 80) is quite high, often as high or higher than his rivals who have higher mean ratings. At the same time, the percent of voters who him a low favorability score (at or below 20) is the highest among candidates who have higher favorability scores than him. The bimodal distribution of preferences over Trump is also indicated by his high bimodal coefficient, which is above the 0.555 threshold for indicating a bimodal distribution. None of the other candidate's coefficients pass this threshold, though Bush approaches it in Iowa and New Hampshire.

5 Validating the Survey & Feeling Thermometer Ratings

Though the surveys of likely Republican voters analyzed in this paper provide rich data on voters' attitudes about candidates in the 2016 Republican primary, there are several potential concerns with these data. The first is the extent to which feeling thermometer ratings predict voters' vote choice. As mentioned, previous work uses feeling thermometer scores for the same purpose (Abramson et al. 1992; Brams and Merrill 1994; Radcliff 1994; Abramson et al. 1995; Regenwetter, Adams and Grofman 2002; Regenwetter, Grofman and Marley 2002; Tideman and Plassmann 2012) and justify this practice given the high (though far from perfect) correlation between voters' vote choice and their feeling thermometer ratings of candidates.

Two other concerns with the use of the survey data are the representativeness of this sample of likely Republican voters and the timing of these surveys relative to when the actual primaries took place. Given the large number of candidates and inability of voters and elites to coordinate around one or two front-runners, many Republican voters were still very uncertain about how they would cast their votes a month or so prior to the primaries when these surveys were conducted, which is partly why the survey firm had voters rate candidates with a feeling thermometer as opposed to indicating their vote choice rank ordering.

One way to address these concerns is to examine how well the feeling thermometers predict the actual percentage of the vote received by each candidate in these races, which

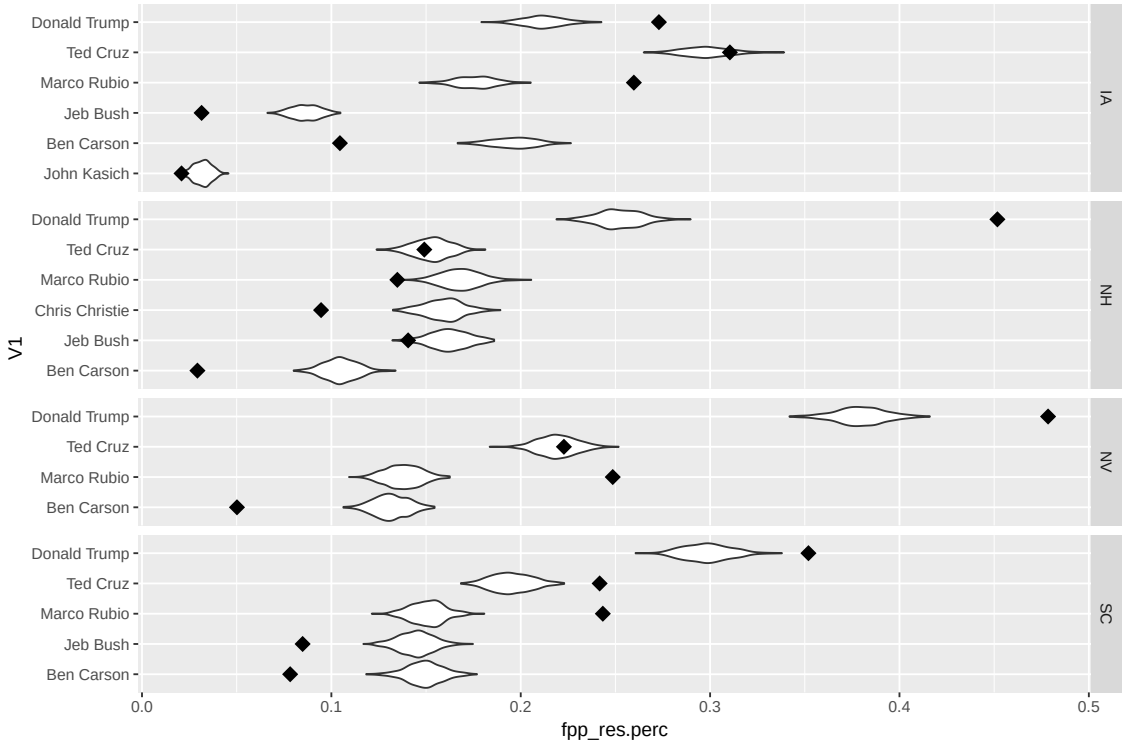

Figure 2: **Actual Vote Share vs. Predicted based on feeling thermometers. Thermometer.** Positive values indicate a candidate performed better than their feeling thermometers would predict. Within each state, candidates are sorted by their vote share.

is illustrated in figures 2 and 3. To do this, we predict what percent of the vote each candidate would receive in a plurality vote under the assumption that respondents would vote for the candidate whom they rate the highest on the feeling thermometer. The predicted vote share is displayed by the white violin plots for each candidate and state in figure 2. The violin plots shows the distributions of the predicted vote share based on a Monte Carlo simulation of ????? simulations. We compare this predicted vote share to the actual vote share that these candidates received in the respective primaries, which is shown by the black diamonds in figure 2. To account for the fact that each survey only asked respondents to rate 6 candidates, we recalculate the actual vote share as the percent of the vote they received among the total voters received by the 6 candidates in each survey. In figure 3, we display the difference between the actual and predicted vote shares under a plurality voting rule. Positive values indicate a candidate performed better than their feeling thermometer score would predict.

Overall, the feeling thermometers systematically under predict Trump's and Rubio's

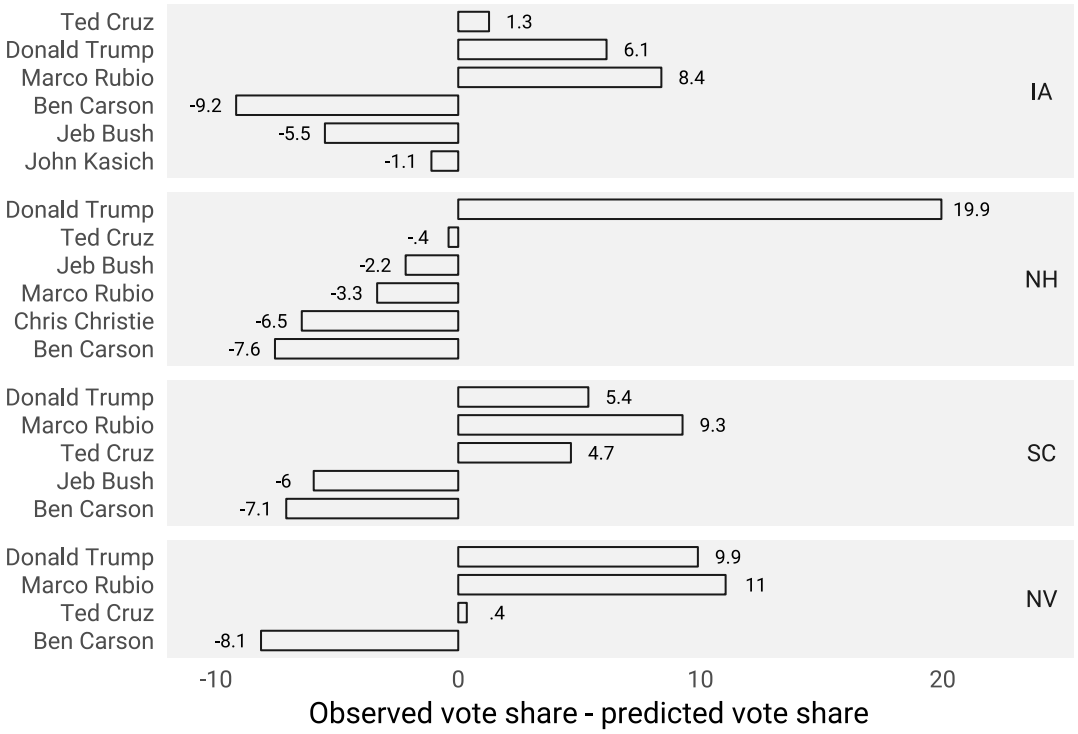

Figure 3: **Difference Between Actual Vote Share vs. Predicted based on feeling thermometers**. Positive values indicate a candidate performed better than their feeling thermometers would predict. Within each state, candidates are sorted by their vote share.

vote shares while over predicting Carson's and Bush's. The feeling thermometers, however, accurately predict Cruz's vote shares in Iowa, New Hampshire, and Nevada. The feeling thermometer ratings are also quite accurate in predicting the order of which candidates received the highest vote share in the actual primaries. Given how much initial primaries can affect vote choice in subsequent contests (Bartels 1988), we give more weight to the results from the Iowa caucuses. For instance, voters in New Hampshire who favored Trump over Cruz and favored another candidate over Trump had incentives to vote strategically for Trump to ensure that Cruz did not win in New Hampshire after winning in Iowa. Republican voters in Iowa, however, had less information available to coordinate strategic voting, which may be why the feeling thermometer ratings are better predictors of actual vote shares there than in some of the other states. And though the predicted vote shares based on feeling thermometers are lower than the actual vote shares for Trump and Rubio (who received the second and third most votes respectively in the actual caucuses), the difference between their predicted vote shares and actual vote shares are quite similar. If anything, using the feeling thermometer ratings to predict voters' preferences over the candidates may actually give Trump an advantage since his lead over Rubio is actually higher in the predicted vote shares than in the actual vote shares.

To further explore whether feeling thermometers are a valid measure of voters' preference ordering in their voting decision, we examine survey results from a 2016 Pew Research Center survey which asked respondents to indicate their first and second vote choice in the Republican primary and to evaluate the candidates on a 101-point feeling thermometer. Thus far, this is the only survey we have been able to access that asks respondents to indicate their vote choice and rate candidates on a feeling thermometer. The survey was conducted from April 5 to May 2, 2016, just prior to when Cruz and Kasich dropped out on May 3 and 4, respectively. All the other candidates had dropped out before the survey began. In examining the data, we find that respondents' vote choice strongly correlated with their feeling thermometer ratings of the candidates. 73% of Republicans who indicated that Trump was their first vote choice also gave him the highest rating on the feeling thermometer. The results are identical among Cruz supporters. Among Kasich supporters, 99.9% rated Kasich higher on the feeling thermometer than the other remaining two candidates. These results are similar to those found by previous studies using feeling thermometers to predict vote choice in US presidential elections and primaries (Abramson et al. 1992; Radcliff 1994; Abramson et al.

1995).

With the Pew survey data, we can also examine the extent to which feeling thermometer ratings predict voters' first and second vote choice since the survey asked respondents to indicate these. To examine this, we calculated the difference in respondents' feeling thermometer ratings of their first and second vote choice. In figure 4, we plot these differences for each of the three candidates who were still in the primary election when the survey was administered. We also plot the differences pooled across all three candidates. Overall, the data show that voters generally rated their first choice higher than their second choice. Among those who indicated that Trump was their first vote choice, 68% gave him a higher rating on the thermometer score than they did their second choice. (Please note that this percentage is lower than the percentage reported above because some respondents indicated that they did not have a second choice.) Among Cruz supporters, 78% gave him a higher feeling thermometer rating than their second vote choice. This number jumps to 91% among Kasich supporters.

To the extent that the relationship between feeling thermometer scores and vote choice ordering from the Pew study applies to our other surveys taken just before the primaries, it suggests that feeling thermometer ratings correlate with vote choice and can even discriminate between individuals' first and second vote choice. Although these correlations are not perfect, they are still quite impressive given the general difficulty of predicting primary voters' vote choice using other variables besides asking them directly whom they would vote for. The results from the Pew survey suggest that feeling thermometer ratings are a weaker predictor of Trump voters than at least Cruz and Kasich voters, which may be why our estimates of the candidates' vote shares based on feeling thermometer ratings under predict Trump's actual vote share. At the same time, we should be careful in over generalizing the results from the Pew study since it was conducted right as Cruz and Kasich were deciding to drop out of the race. Thus, many of the respondents who indicated that they would vote for Trump may have been basing that choice off of their perception of who would likely win the nomination—a common factor in primary voters' vote choice (Bartels 1988)—as opposed to other important factors in their vote choice. This concern highlights the importance of focusing on early primaries and Iowa in particular, the state in which feeling thermometer ratings are the strongest predictor of vote choice as displayed in figures 2 and 3.

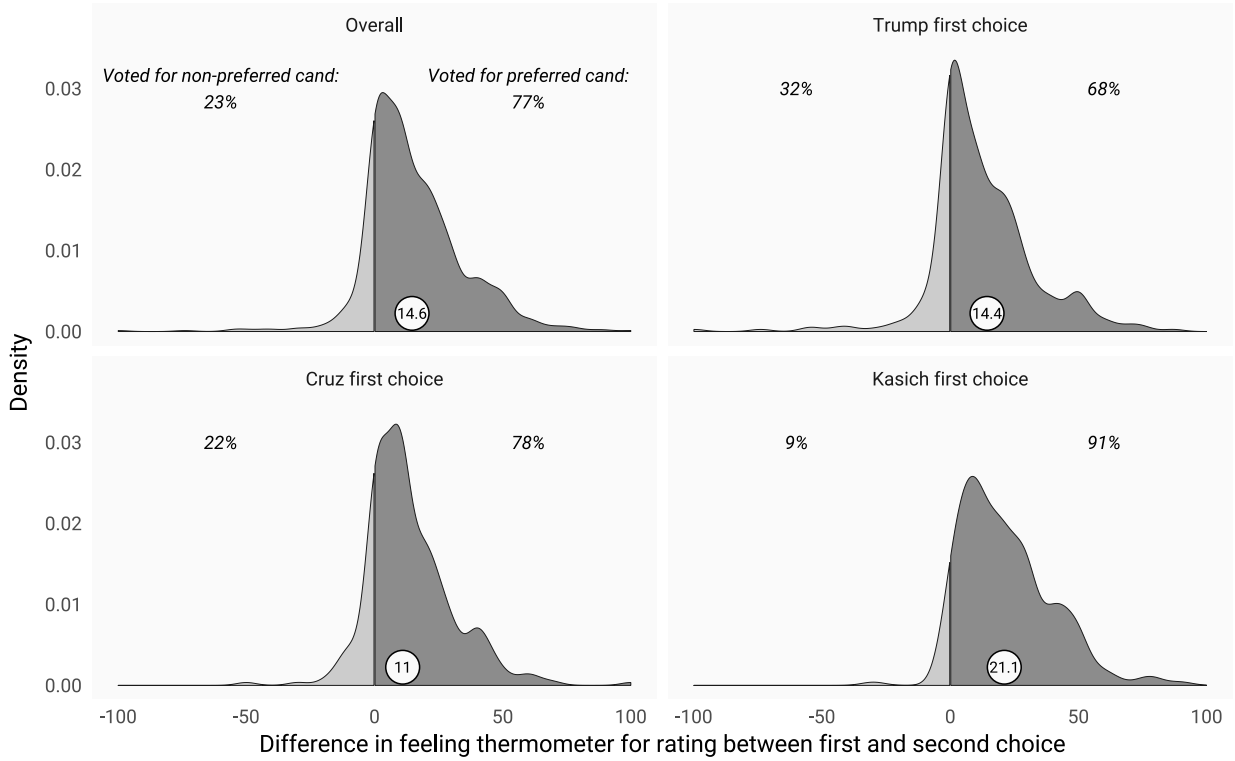

Figure 4: **Difference in Feeling Thermometer Rating Between First and Second Vote Choice**. The top left facet shows the overall distribution in the difference in feeling thermometer evaluations between a respondents first and second choice in the Republican primary. Any positive value (shaded dark in the distributions above) indicate a respondent indicates a respondent gave a higher evaluation to their first choice candidate than their second. Labels within points report the mean of each distribution. Labels above report the percentage of respondents who voted for their preferred on non-preferred candidates.

6 Who Wins Under Different Voting Rules?

We now turn to analyzing how the election results in the 2016 Republican primaries in Iowa, New Hampshire, South Carolina, and Nevada could have potentially changed under different voting rules. To help evaluate these outcomes, we begin by examining whether a Condorcet winner exists in each state. In social choice theory, this is an important benchmark in determining whether an election result is in line with the majoritarian ethos of democratic governance. According to many social choice theorists, if a Condorcet winner exists, they should ideally be the winner.

To test whether this occurred in each state, we predict how each candidate would fare in a two-candidate election against each of the other candidates based on respondents' feeling thermometer ratings. The assumption underlying this analysis is that respondents would vote for whichever candidate they rate higher on the feeling thermometer score. In the case of tied thermometer scores, we randomly select one of the candidates as that respondent's choice. To account for sampling error, we bootstrap the data and estimate the percent of times that each candidate beats each of the other candidates. The results from this analysis are in table 2. The cell entries display the percentage of victories enjoyed by the candidates indicated in the row over the candidate indicated in the columns. The last three columns on the right indicate the rank orderings of the candidates based on either the predicted Condorcet winners (using the feeling thermometer scores), the predicted plurality winners (also using the feeling thermometer scores), or the actual plurality winners in the primaries.

The pairwise comparisons in table 2 reveal some interesting findings. First, there is a clear Condorcet winner and rank-ordering in each state as indicated by the corresponding rankings column on the right side of 2. In other words, we do not find any cycling in the results. In Iowa, we predict that Cruz would win 100% of the time against each of the other candidates. Rubio would do the same in New Hampshire. Trump nearly performs just as strongly in Nevada, though we predict that he would beat Cruz 99% of the time as opposed to 100% of the time. Finally, Cruz is the Condorcet winner again in South Carolina beating all of the other candidates at least 99.3% of the time.

The second important finding, which we anticipated given the high number of candidates and their close evaluations, is that the Condorcet winner is not always the winner in these states under a plurality rule. In two states, Iowa and Nevada, the Condorcet winner was chosen under plurality rule. But in the other two, New Hampshire and South Carolina, the Condorcet winner did not win. Again, from a democratic perspective, this

							Rankings		
							Predicted		Actual
			Iowa				Cond.	Plur.	Plur.
	Kasich	Bush	Trump	Rubio	Carson	Cruz			
Cruz	100	100	100	100	100		1	1	1
Carson	100	100	100	94.34		0	2	3	4
Rubio	100	100	100		5.66	0	3	4	3
Trump	100	100		0	0	0	4	2	2
Bush	100		0	0	0	0	5	5	5
Kasich		0	0	0	0	0	6	6	6
			New Hampshire						
	Trump	Bush	Carson	Cruz	Christie	Rubio			
Rubio	100	100	100	100	100		1	2	4
Christie	99	100	100	97.34		0	2	3	5
Cruz	100	97	100		2.66	0	3	5	2
Carson	94	96.3		0	0	0	4	4	6
Bush	52		3.7	3	0	0	5	6	3
Trump		48	6	0	1	0	6	1	1
			Nevada						
	Bush	Christie	Carson	Rubio	Cruz	Trump			
Trump	100	100	100	100	99		1	1	1
Cruz	100	100	100	100		1	2	2	3
Rubio	100	100	97.6		0	0	3	3	2
Carson	100	100		2.4	0	0	4	4	4
Christie	100		0	0	0	0	5	5	6
Bush		0	0	0	0	0	6	6	6
			South Carolina						
	Christie	Bush	Trump	Carson	Rubio	Cruz			
Cruz	100	100	99.6	100	99.3		1	2	3
Rubio	100	100	61.6	94.6		0.7	2	3	2
Carson	100	100	85		5.4	0.7	3	4	5
Trump	100	100		15	38.4	0.4	4	1	1
Bush	96.67		0	0	0	0	5	5	4
Christie		3.33	0	0	0	0	6	6	6

Table 2: **Predicted Winners in Pairwise Comparisons**. Cell entries display the percentage of victories enjoyed by the row candidates over the column candidates in a pairwise comparison based on likely Republican voters' feeling thermometer ratings of each candidate. * indicates Condorcet winners. ^ indicates actual and predicted plurality winners.

is a troubling result and likely helps explain why so many Republicans were dissatisfied with the results of the primary.

A third interesting finding is how different the rank orderings of the candidates are in the pairwise comparisons relative to those under the plurality rule. Trump, in particular, stands out as performing much better in the plurality rankings than the Condorcet ones. For example, in Iowa, we predict that Trump would win second place under plurality rule, which he also did in the actual election. But in the pairwise comparisons, he ranks fourth. And though the feeling thermometers potentially overrate Carson's performance in the election, Trump is handily beaten by Rubio in a pairwise comparison, losing on average with just 43% of the vote. Yet, under plurality rule, he squeaks by Rubio, achieving second place.

This pattern of Trump performing much better under the plurality rule than in the pairwise comparisons, happens in New Hampshire (where he goes from first to last) and South Carolina (where he goes from first to fourth). Though we are cautious in over interpreting the results from New Hampshire given the large differences in our predicted plurality vote shares and the actual ones, the results in Iowa and South Carolina, where our predictions are more accurate, are quite similar.

Nevada is the state that stands out in the sense that the rankings are very similar across the different voting rules. There, Trump is both the Condorcet winner and plurality winner. In fact, the rank orderings are the same between these two methods of predicting candidates' performance.

The results alone from table 2 suggest that the voting rules used in these four primaries biased the election results away from candidates with more majoritarian support. To further examine the role of voting rules on the electoral outcomes in these primaries, we use the thermometer scores to predict who would win under different voting rules. As with 2, we bootstrap the survey results to estimate the percent of time that each candidate would win using different voting rules under the assumption that respondents' rank ordering of candidates on the feeling thermometer represents their vote choice rank-ordering. To predict the winners under approval voting, we assumed that respondents would vote for all candidates they rated above a certain threshold on the feeling thermometer scores. In this case, we chose ratings of 90, 75, and 50.A threshold at a rating of 50 is rather low given that most candidates had average ratings above this. Thus, we suspect that most voters would have a higher threshold than this. If a respondent did not rate any candidates above that threshold, we assumed they would just vote for

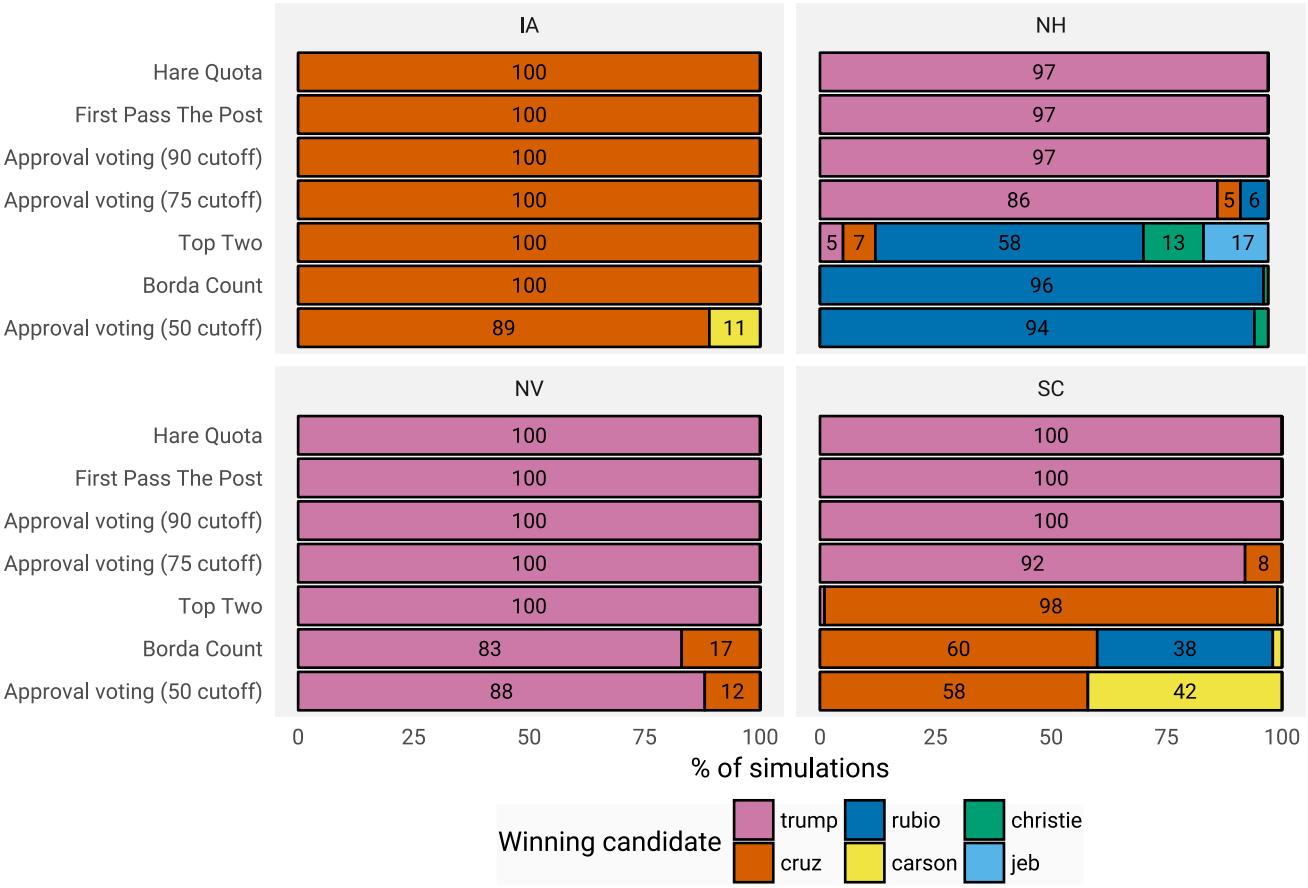

Figure 5: **Percent of Times Candidates Win Under Different Voting Rules**. This figure indicates the percent of the time that each candidate would have won under different voting rules.

their highest rated candidate.In subsequent versions of this paper, we plan to use additional criteria to estimate for whom respondents would vote under approval voting. For instance, we could assume that respondents would vote for their top two or three picks, as long as their ratings were above a certain threshold. We could also identify large gaps in the ratings, and assume that respondents would vote for anyone above that gap. We could also randomize the threshold across respondents. The results from this analysis are presented in figure 5.

The results in figure 5 are missing two voting rules that we described earlier in the paper. The first is the Coombs rule, which we have not yet been able to calculate. The second is the Black rule, which picks the Condorcet winner, if one exists, and then uses the Borda rule. As demonstrated in table 2, there was a clear Condorcet winner in each

state. Thus, under the Black rule, Cruz is the winner in both Iowa and South Carolina; Rubio is the winner in New Hampshire; and Trump is the winner in Nevada. This is a very different outcome than what we predict under plurality rule, where Trump wins three of the four elections instead of just one.

Overall, we find that Cruz dominates in Iowa while Trump dominates in Nevada, regardless of the voting rule employed. In New Hampshire, Trump and Rubio alternate as the winners, with Trump winning under plurality, Hare, and approval voting with a high threshold; while Rubio is the likely winner under a top-two runoff, Borda count, low-threshold approval voting, and the Black rule. Finally, in South Carolina, Trump and Cruz are the primary winners in a similar way that Trump and Rubio were in New Hampshire. Again, Trump wins under plurality, Hare, and approval voting with a high threshold. Cruz wins under top-two runoff, Borda, and low-threshold approval, through Rubio and Carson have a decent probability of winning under Borda (prob. = .38) and low-threshold approval (prob. = .42), respectively.

The results in figure 5 also demonstrate the shortcomings of different voting rules under the normative arguments that Condorcet winners should be selected when they exist. In New Hampshire and South Carolina, several of the rules fail to select the Condorcet winner. This is the case for Hare, plurality, and high-threshold approval voting, which is a more realistic cutoff for predicting approval voting choices than a feeling thermometer of just 50, which is supposed to indicate indifference between favorable and unfavorable feelings.

7 Discussion & Conclusion

In this paper, we examine how voting rules (such as plurality, top-two runoff, approval voting, etc.) potentially affected the results of the 2016 Republican presidential primaries in Iowa, New Hampshire, South Carolina, and Nevada, the first four states that had primaries and thus set the stage for the remainder of the primaries, including leading several prominent candidates to drop out of the running. Our results suggest that the Condorcet winner (i.e., the candidate who could beat all other candidates in a pairwise comparison) did not win in two of the four states—New Hampshire and South Carolina. In addition, we find that different candidates would have won in those two states under several different voting rules—top-two runoff, Borda, and Black. The latter rule is also considered to be one of the least susceptible to a variety of election paradoxes that under-

mine the democratic intentions of even holding elections. If these other rules were implemented instead of the current practice of determining "winners" by plurality rule, we predict that Trump may have only won in one state (Nevada) instead of three (Nevada, New Hampshire, and South Caroline). In addition, his performance in Iowa would have also been diminished under pairwise comparisons, where he finished fourth instead of second.

Though we cannot know the counterfactual of how the 2016 primaries would have played out under these different rules, it is not hard to imagine that such different outcomes in the first four states in the primaries may have set the stage for a different candidate to have won, especially if other states also used other voting rules that favored candidates with more majoritarian support. Though there are good reasons to be skeptical of our ability to predict who would have won in the three states that followed Iowa, just the different results in Iowa (moving from second to fourth) could have set the stage for an alternative outcome in the subsequent primaries. Though Trump only won second in Iowa, his performance exceeded expectations and became the major story from that race, likely improving his performance in the races that followed and New Hampshire in particular, where Cruz had less support than he did in other early primary states.

This finding has important implications for party officials as they seek to optimize their parties' primary process. As a potential reform, changing the voting rules is likely to encounter less resistance from the party rank-and-file than attempts to empower party officials with more influence over the selection process, such as through super delegates. Indeed, reforming the voting rules would actually empower voters more than continuing to use plurality voting rules since it is much more susceptible to the Condorcet winner paradox than other rules.

The implications of these results extend beyond the 2016 Republican presidential primary and to any primary that features multiple, viable candidates. The norm in most of the elections in the US is to use plurality voting to determine the winner. But as we show here, this process can result in the nomination of candidates who do not have broad support among party voters. As mentioned in our introduction, many primaries at the Congressional level feature multiple viable candidates.

Finally, these findings also have implications for how we should interpret the meaning of Trump's nomination. It is common for political observers to suggest that his win means that Republican voters (as a whole or majority) support Trump's agenda and

mode of operation. Here is just one example from a recent article in the *Washington Post*:

> Even those Republicans who hesitate to embrace Trump have come to grips with the fact that voters chose this mode of operation — and no longer expect him to change course.
>
> "They picked the person that had the least government experience, and was a business guy, and who bragged about saying, 'I'm completely out of the mainstream,' and they said, 'That's who I want,' and they got that the first year," Sen. James Lankford (Okla.), who supported other Republicans in the 2016 primaries, said on the eve of the GOP retreat at the Greenbrier resort here.[12] February, 1.

In the excerpt above, both the reporter and the US Senator incorrectly assert that Trump's nomination and electoral victory mean that voters chose and therefore implicitly approve of his approach to governance. Besides ignoring that electoral choices are often multi-dimensional, these assertions by the reporter and US Senator also ignore how the system used to count and aggregate votes can change the outcome of elections even when voters' preferences are held constant. This is important for claims about so-called electoral "mandates" and for elected officials' and candidates' decisions about updating their behavior in response to election results. For instance, future candidates who think Trump is an example of a successful approach to winning elections should reconsider whether such an approach would have worked if Trump faced one strong primary candidate instead of multiple or if voter's preferences were aggregated differently, as we examine here.

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